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REPORT TITLE:

Electromagnetic Waves Modeling

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I. Introduction

The finite difference time domain (FDTD) method is a power full method for modeling and simulating some various physical phenomenon.

The finite difference time domain (FDTD) method is a full wave, dynamic, and powerful solution tool for solving Maxwell's equations, introduced by K.S. Yee in 1966 [7]. The algorithm involves direct discretization of Maxwell's equations by writing the spatial and time derivatives in a central finite difference form.

In this paper, the FDTD method will be applied to simulate electromagnetic waves propagating in different mediums (free space, dielectric). The method is based on the discretization of the Maxwell's equations by applying the central difference approximations.

II. Maxwell's equations

The time-dependent Maxwell's curl equations in homogeneous dielectric medium [2] ($\varepsilon=\varepsilon_0\varepsilon_r$, $\mu=\mu_0$, $\mu_r=1$) are

$$\frac{\partial \mathrm{E}}{\partial t} = \frac{1}{\epsilon_0 \epsilon_r} \nabla \times \mathrm{H} \tag{1}$$

$$\frac{\partial \mathrm{H}}{\partial t} = \frac{1}{\mu_0} \nabla \times \mathrm{E} \tag{2}$$

E and H are vectors in three dimensions. The constants ε_0 and μ_0 are know as the permittivity and permeability of free space and ε_r is the relative permittivity of the material.

Equations (1) and (2) in one-dimensional will became

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0} \frac{\partial H_y}{\partial z} \tag{3}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_x}{\partial z} \tag{4}$$

These equations represent a plane wave with the electric field oriented in the x-direction and the magnetic field oriented in the y-direction and traveling in z-direction.

Using the central difference approximations in temporal and spatial derivatives, these equations will became

$$\frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t} = -\frac{1}{\varepsilon_0}\frac{H_y^{n}(k+1/2) - H_y^{n}(k-1/2)}{\Delta x} \tag{5}$$

$$\frac{H_y^{n+1}(k+1/2) - H_y^{n}(k+1/2)}{\Delta t} = -\frac{1}{\mu_0}\frac{E_x^{n+1/2}(k+1) - E_x^{n+1/2}(k)}{\Delta x} \tag{6}$$

in the equations above, n is the time index and k is the spatial index, which indexes times $t = n\Delta t$ and positions $z = k\Delta z$, or times $t = (n\pm 1/2)\Delta t$ and positions $z = (k\pm 1/2)\Delta z$. The time index is written as a superscripts, and the spatial index is within brackets.

Equations (5) and (6) can be rearranged to be used in computer program as $E^{n+1/2}{}_x(k)$ and $H^{n+1}{}_y(k+1/2)$, corresponding the $E_x(t+ \Delta t/2, z)$ and $H_y(t+\Delta t, z+\Delta z/2)$.

These two equations above will rearranged as

$$E_x^{n+1/2}(k) = E_x^{n-1/2}(k) - \frac{\Delta t}{\varepsilon_0 \cdot \Delta x}\left[H_y^{n}(k+1/2) - H_y^{n}(k-1/2)\right] \tag{7}$$

$$H_y^{n+1}(k+1/2) = H_y^{n}(k+1/2) - \frac{\Delta t}{\mu_0 \cdot \Delta x}\left[E_x^{n+1/2}(k+1) - E_x^{n+1/2}(k)\right] \tag{8}$$

These above equations look very similar. Only ε_0 and μ_0 differ by several orders of magnitude:

$\varepsilon_0 = 8.85\text{x}10^{8}$ F/m,

$\mu_0 = 4\pi\text{x}10^{-7}$ H/m.

Therefore, E_x and H_y will differ also by several orders of magnitude. Numerical error is minimized by making the following change of variables as

$$\tilde{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}}E \tag{9}$$

which bring the field quantities to similar levels. Substituting this into equations (7) and (8) gives

$$\tilde{E}_x^{n+1/2}(k) = \tilde{E}_x^{n-1/2}(k) - \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \frac{\Delta t}{\Delta x}\left[H_y^n(k+1/2) - H_y^n(k-1/2)\right] \tag{10}$$

$$H_y^{n+1}(k+1/2) = H_y^n(k+1/2) - \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \frac{\Delta t}{\Delta x}\left[\tilde{E}_x^{n+1/2}(k+1) - \tilde{E}_x^{n+1/2}(k)\right] \tag{11}$$

The similar procedure (discretization) will be applied for the two-dimensional [2]. We will get three fields E_z, H_x, and H_y, or E_x, E_y, and H_z depends to the polarization considered (Transverse Magnetic (TM) and Transverse Electric (TE) respectively).

III. Accuracy and Stability

The order error of the central difference approximation indicate that using a too larger Δz would lead to numerical inaccuracies [1]. Intuitively we can reason that enough sampling points must be taken to ensure adequate representation of a signal and we must use more sampling points for a highly variable signal. Therefore, in FDTD Δz depends on frequency. A good rule of thumb is that $\Delta z < \lambda/20$ for the highest frequency of interest.

Another concern is stability of the simulation. An electromagnetic wave cannot go faster than the speed of light. In free space and one dimensional case, an electromagnetic wave travels a distance of one cell in time $\Delta t = \Delta z/c_0$, where c_0 is the speed of light in free space. This limits the maximum time step.

Once the cell size Δz is chosen, then the time step Δt is determined by

$$\Delta t = \frac{\Delta x}{2 \cdot c_0} \tag{12}$$

Therefore

$$\frac{1}{\sqrt{\varepsilon_0 \mu_0}} \frac{\Delta t}{\Delta x} = c_0 \cdot \frac{\Delta x / 2 \cdot c_0}{\Delta x} = \frac{1}{2} \tag{13}$$

IV. Simulation

IV-1. One-dimensional (1-d)

a. free-space

An electromagnetic pulse is radiated from a source located in free space. The source waveform is a Gaussian shaped pulse, injected at the centre of the array used to store the pulse in space. This is achieved by repeatedly updating the E-field pulse value at the source location; the value at the other locations are computed using the update equations. Figure shows the simulation result as time progresses. The E-field is seen to propagate away from the source to the left and to the right.

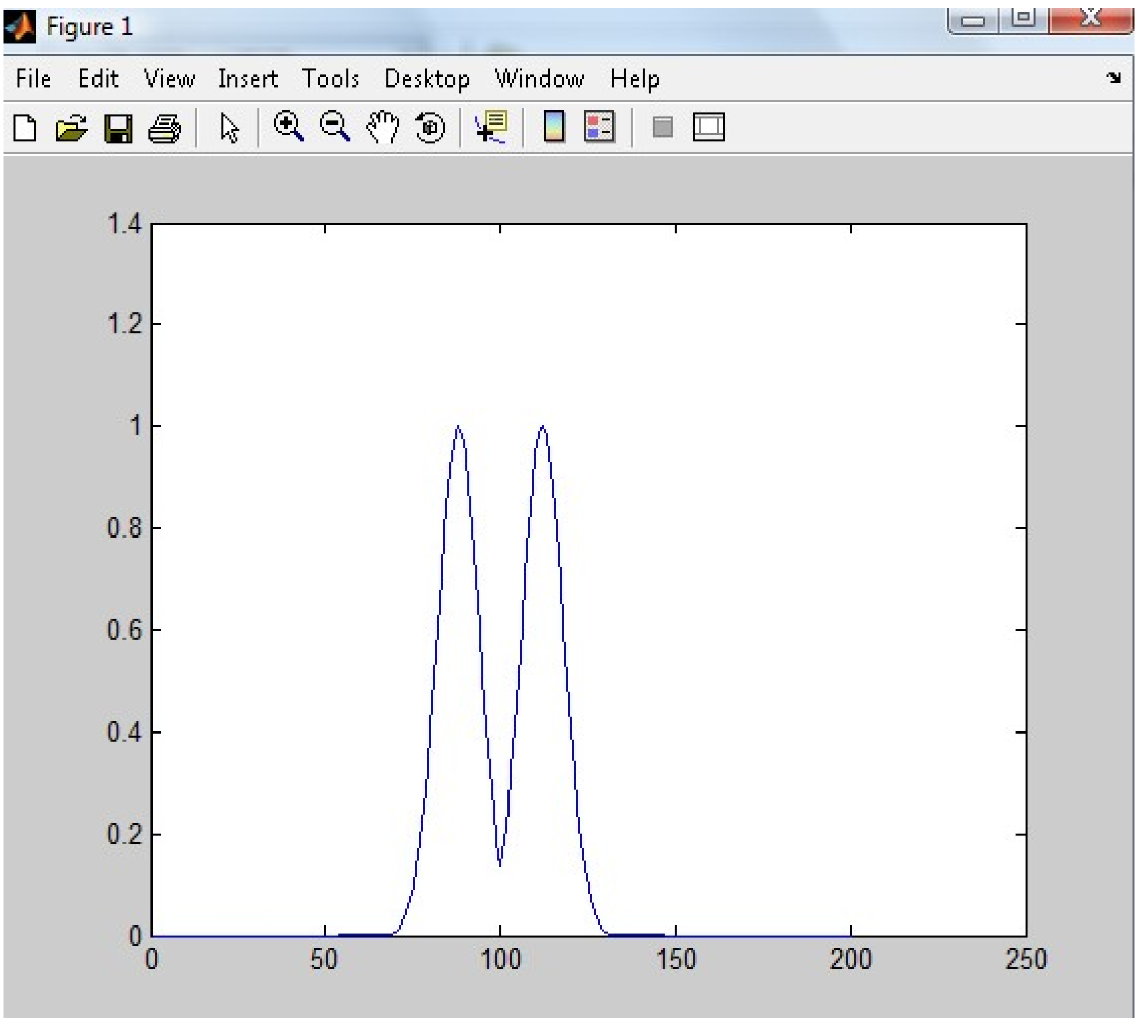

Figure 1 : E_z propagation in free-space

b. Wave hitting a dielectric medium

In this case, the plane wave is traveling in free space (medium 1) strikes a dielectric medium (medium 2), as is illustrated in figure, which shows a source in free space on the left side, and a dielectric slab on the right.

When the wave strikes the interface, a fraction of the incident wave is reflected, and a fraction is transmitted into the medium2. The amplitude of the reflected and transmitted waves, relative to the incident wave, are described by the reflection coefficient ρ and the transmission coefficient τ, which relate the amplitude of E field waves.

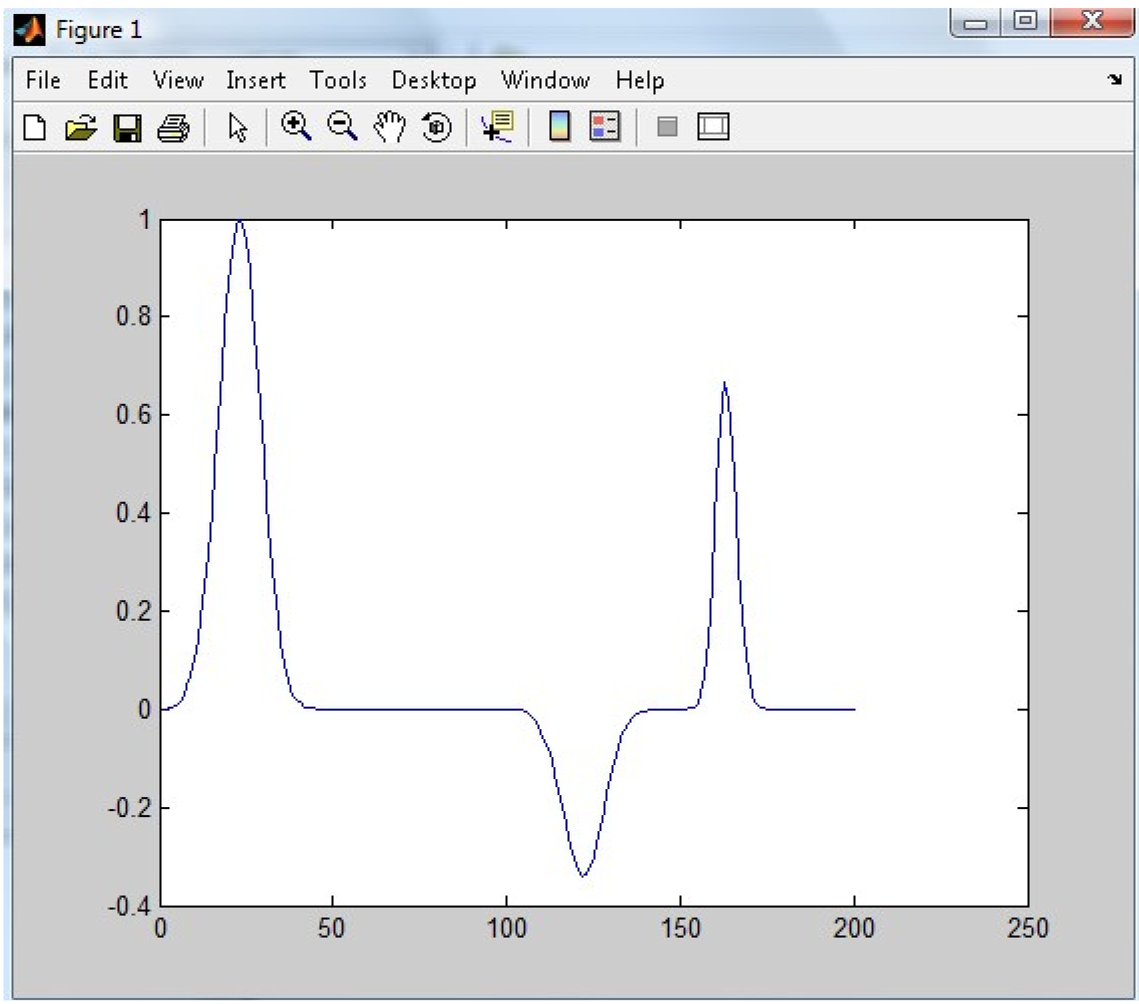

Figure 2 : E_z hitting a dielectric

c. Removing the unwanted reflection at the boundary-"The Absorbing Boundary Condition (ABC)"

At the edge of the problem space, we will not have the value to one side, but we know there are no sources outside the problem space. The wave travels *Δz/2 (=c_0Δt)* distance in one time step, so its takes two times steps for a wave front to across one cell.

Suppose we are looking for a boundary condition and the end where k=1. Now if we write the E-field at k=1 as

$$E_x^n(1) = E_x^{n-2}(2)$$

Then the fields at the edge will not reflect. This condition must be applied at both ends.

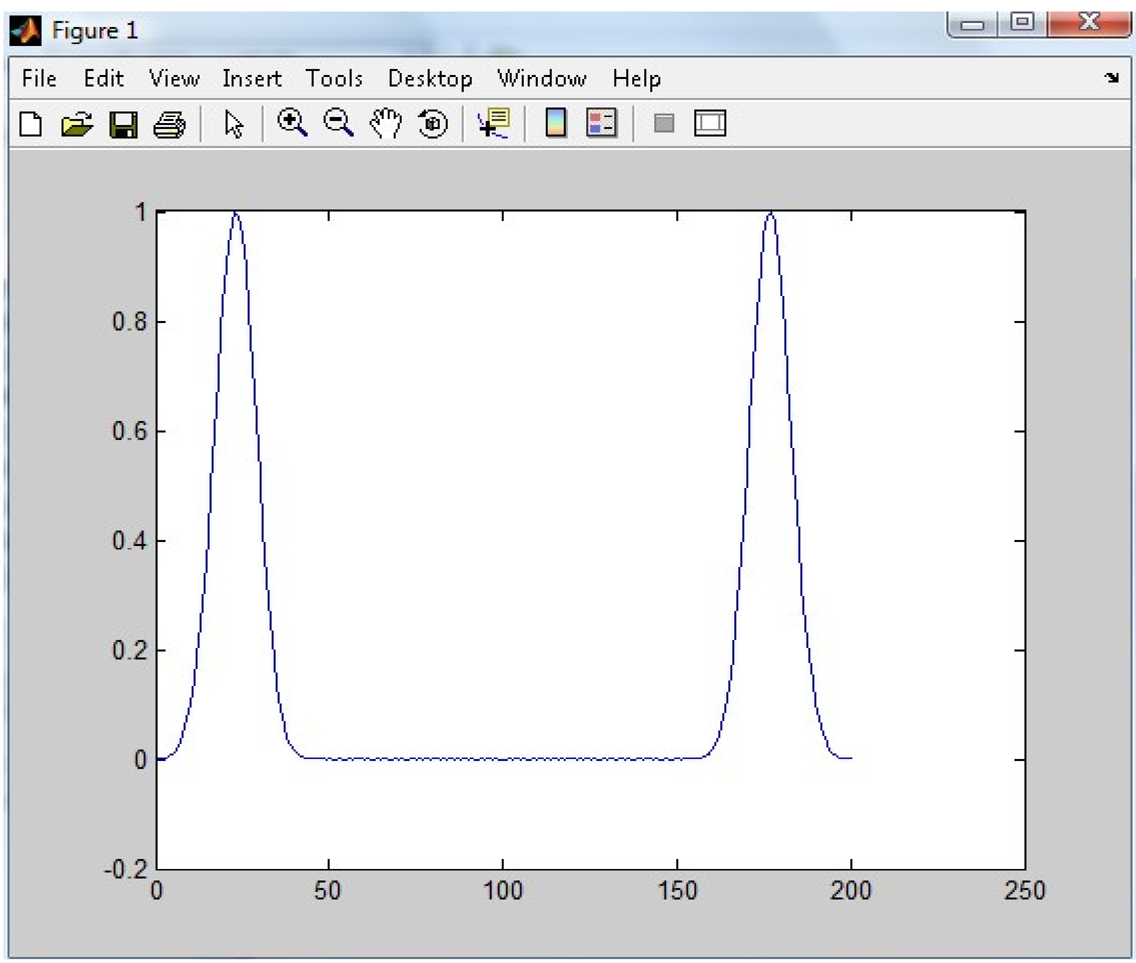

Figure 3 : E_z propagating in free-space with Absorbing Boundary Condition applied at both ends

IV-2. Two-dimensional (2-d)

a. Wave hitting a dielectric surface

In two-dimensional, we have two choices:

-The transverse magnetic (TM) mode, which is composed of E_z, H_x, and H_y,

-The transverse electric (TE) mode, which is composed of E_x, E_y, and H_z.

The simulation below is achieved with TM mode wave propagation ($\partial/\partial z = 0$, $E_x = 0$, $E_y = 0$, and $H_z = 0$).

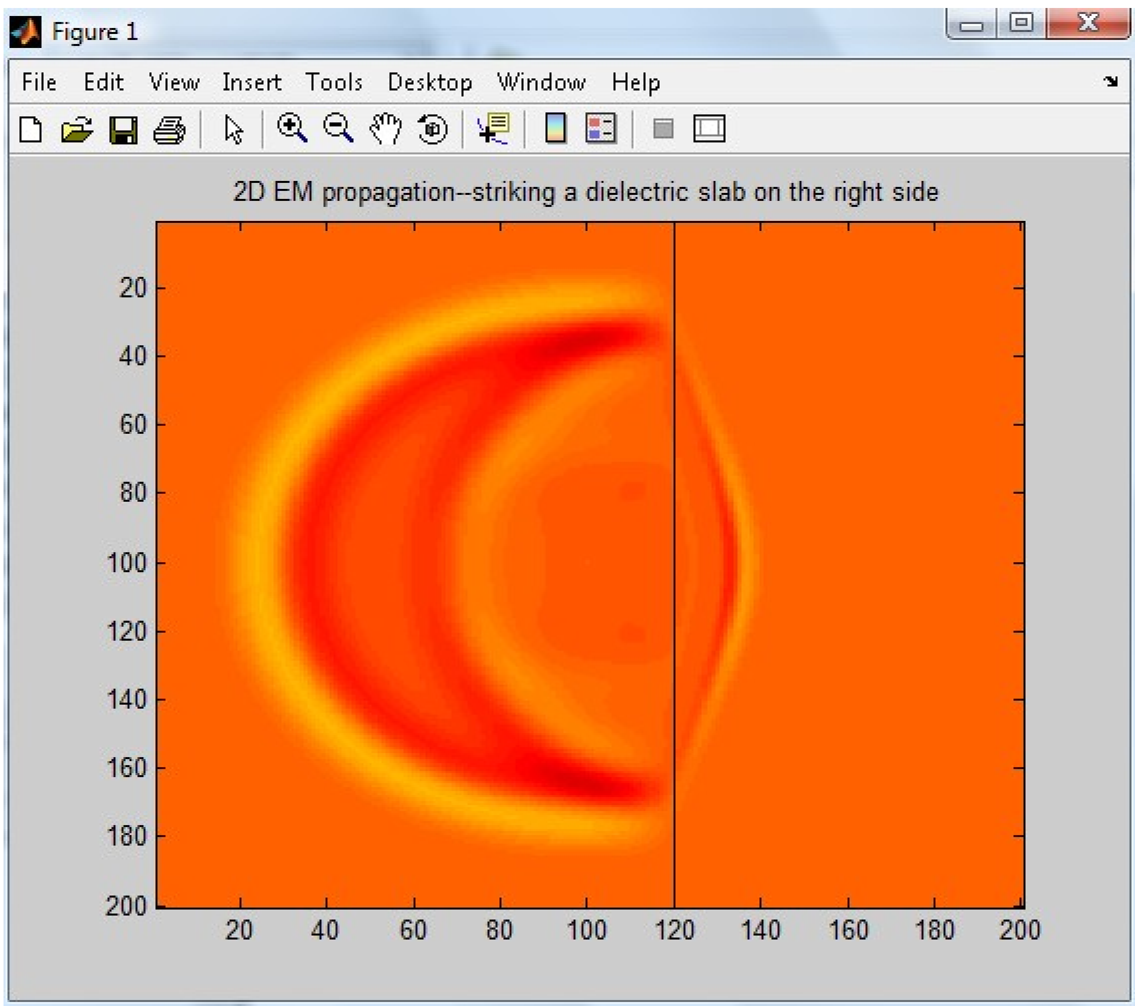

Figure 4 : wave propagating in free-space and hitting a dielectric at right in 2-d

V. Conclusion

In this paper, an electromagnetic waves are simulated by the Finite Differential Time Domain (FDTD) Method in one-dimensional and two-dimensional. Some particularities has been applied like the reflection between free-space and dielectric medium, and the absorbing boundary conditions (ABC).

The results obtained reflect the theoretical behavior of the electromagnetic waves.

The next study will be done with three-dimensional and will integrate more complicate mediums.

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