On the Performance of Sequences for Uplink QS-CDMA System

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***Abstract*— Loosely Synchronized (LS) sequences having Interference Free Window (IFW) are considered to be good candidates for QS-CDMA systems. The recent LS sequence constructions have generated very small sets of sequences exhibiting IFW, thus making them suitable for lightly loaded QS-CDMA. In this paper, it is shown that under certain scenarios for generalized lightly loaded QS-CDMA, there exist traditional sequences, that give either comparable performance or outperform LS sequences, in terms of cross-correlation values. We have demonstrated our results, using short length sequences of length L=31, 32and 38. We have shown that for a QS-CDMA system with 15 users, and with chip delays equal to IFW of LS sequences, subsets of WH and Oppermann sequences outperform family of LS sequences. Also, either they give comparable performance or outperform LS sequences, when compared for 8 user QS-CDMA system, with a single or two user delays, 1-3 chips outside the IFW of LS sequences. Moreover, to demonstrate our results we have compared BER of uplink QS-CDMA system in AWGN channel, using different sequence subsets and different scenarios.**

Index Terms

Bit Error Rate (BER), Interference Free Window (IFW), Loosely Synchronized (LS), Multiple Access Interference (MAI), Signal-to-Noise Ratio (SNR), Quasi-Synchronous CDMA (QS-CDMA), Zero Correlation Zone (ZCZ).

I. INTRODUCTION

The interference limited capacity of traditional Code Division Multiple Access (CDMA) has generated great deal of interest in Quasi-Synchronous Code division Multiple access (QS-CDMA) technology [1]. In QS-CDMA, chip synchronization is maintained by allowing user delays within certain limit. Thus, a QS-CDMA system can be classified as a synchronized system with some synchronization errors [2]. For QS-CDMA system, sequences with Interference Free Window (IFW), also known as Zero Correlation Zone (ZCZ) sequences are considered to be good candidates. These ZCZ sequences exhibit IFW within certain chip delays, and hence can achieve low interference and low error rates [3]. There has been considerable research efforts to construct different families of ZCZ sequences. These research efforts have resulted in the construction of Loosely synchronized (LS) sequences, which exhibit ZCZ within certain delay profile. In other words, LS codes exhibit an IFW, where the off peak aperiodic correlation values become zero [4].

In [5], first fifteen sequences of different orthogonal polyphase sequence sets were compared with two different constructions of polyphase LS sequences. It is shown in [4] that with the existing construction methods the number of sequences exhibiting IFW is very low. Due to this reason, for a particular LS sequence set of family size M and length N, there exist small set of sequences exhibiting IFW. This makes them suitable for lightly loaded QS-CDMA systems. Moreover, these sequences also have ZCZ bound [6]:

$$Z \leq \frac{N}{M} - 1. \tag{1}$$

This bound limits the family size M of LS sequences exhibiting ZCZ.

In this paper, LS sequence set performance for QS-CDMA system is analyzed for different chip delays. LS sequences are compared with optimized sequence subsets of traditional sequences like Oppermann and Walsh Hadamard (WH) sequences. Average Mean Square Cross-Correlation (MSCC) and Out-of-phase average Mean Square Auto-Correlation (MSAC) values of these sequence sets for QS-CDMA system are presented. Moreover, uplink QS-CDMA system with Quadrature Phase Shift Keying (QPSK) modulation, operating in Additive White Gaussian Noise (AWGN) Channel is considered. We have generated Bit Error Rate (BER) results for different sequence sets, with 8 and 15 users using QS-CDMA system, operating under different chip delay conditions.

This paper is organized as follows. Section II, defines mean square correlation functions and optimized subsets of sequences. In Section III, some aspects of QS-CDMA capacity, related to cross-correlation values of sequences are discussed. Simulation model and results are presented in Section IV, and Section V, concludes the paper.

†This work was supported by Early Stage Researcher (ESR) Programme of Marie Curie Actions (FP6/EST3).

II. Mean Square Correlation Functions and Optimized Subsets

The average BER of asynchronous CDMA is related to average interference caused by aperiodic cross-correlation properties of the sequence sets used in the system. It is therefore more practical to consider average mean square values of cross-correlation and auto-correlation [7]. The average MSCC measure for a sequence set of size M and length N for a traditional CDMA system is defined as [7]:

$$R_{cc} = \frac{1}{M(M-1)} \sum_{k=1}^{M} \sum_{j=1, j\neq k}^{M} \sum_{\tau=1-N}^{N-1} \mid C_{k,j}(\tau) \mid^2, \quad (2)$$

where $C_{k,j}(\tau)$ is the aperiodic cross-correlation measure between two sequences c_k and c_j of length N and is defined as [3]:

$$C_{k,j}(\tau) = \begin{cases} \frac{1}{\sqrt{E(c_k)E(c_j)}} \sum_{n=0}^{N-\tau-1} c_k(n) c_j^*(n+\tau), & 0 \leq \tau \leq N-1 \\ \frac{1}{\sqrt{E(c_k)E(c_j)}} \sum_{n=0}^{N+\tau-1} c_k(n-\tau) c_j^*(n), & 1-N \leq \tau < 0 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where '*' represents complex conjugate of the sequence, τ represents relative shift between the sequence pairs and $E(c_k)$ represents expected value of the sequence and is given as [3]:

$$E(c_k) = \sum_{n=0}^{N-1} \mid c_k(n) \mid^2 . \quad (4)$$

Since, QS-CDMA maintains chip synchronization by allowing user delays within certain limit, therefore average MSCC measure for a QS-CDMA sequence set will be slightly modified. For a QS-CDMA system, Eq. 2 will be modified to become:

$$R_{cc} = \frac{1}{M(M-1)} \sum_{k=1}^{M} \sum_{j=1, j\neq k}^{M} \sum_{\tau=1-L}^{L-1} \mid C_{k,j}(\tau) \mid^2, \quad (5)$$

where L is the delay limit for a QS-CDMA system. For example, if a QS-CDMA system has delay limit of 3 chips then $L = 4$. If delay limit is increased or decreased to further chip delays, then those delays will change the value of L.

In case of average MSAC for sequences $c_1......c_k$ with set size M and length N, we calculate out-of-phase average MSAC and is given by [7]:

$$R_{ac} = \frac{1}{M} \sum_{j=1}^{M} \sum_{\tau=1-N, \tau\neq 0}^{N-1} \mid C_{jj}(\tau) \mid^2, \quad (6)$$

where $C_{jj}(\tau)$ is cross-correlation of sequence c_j with itself. Like average MSCC measure, out-of-phase average MSAC for QS-CDMA will be modified to become:

$$R_{ac} = \frac{1}{M} \sum_{j=1}^{M} \sum_{\tau=1-L, \tau\neq 0}^{L-1} \mid C_{jj}(\tau) \mid^2, \quad (7)$$

where L is again the chip delay limit for a QS-CDMA system.

A) WH and Oppermann Sequences

In this paper, we have used three different sequence families for comparison. First sequence family is traditional WH. We have considered WH set of Length $L = 32$ and optimized subset sizes of $M = 8$ and $M = 15$. Since, there is numerous literature available for the construction of WH sequences. For this reason, we will not discuss this sequence family construction. Our second sequence family is Oppermann sequences, and Oppermann sequence sets exhibit wide range of correlation properties, when slight changes are made in their parameter values (m, n, p). The v^{th} element $u_{k,v}$ of a sequence from Oppermann set is given by [7]:

$$u_{k,v} = (-1)^{k(v+1)} \exp[\frac{j\pi(k^m(v+1)^p + (v+1)^n)}{N}], \quad (8)$$

where $0 \leq v \leq N-1$ and $(m, n, p) \in$ R. In this paper, we will consider Oppermann sequences with $L = 31$, $M = 8$ and $M = 15$, using $(m, n, p) = (1.0038, 1.2971, 1)$. This particular set is optimized in terms of cross-correlation values [7].

B) LS Sequences

LS codes can be constructed by generating orthogonal complementary codes of length N and applying them to a WH matrix $P \times P$. Generation of LS code sets are discussed explicitly in [4], [5] and [8]. In this work, we have used LS code set of (N, P, W_0)=$(4, 8, 3)$ and $L = 38$, where N is the length of original complementary pair and its mate, P represents WH matrix $P \times P$, W_0 is the length of IFW and L is the total length of the sequence including IFW.

To have LS code family of length 38 (including IFW zeros), we can take a complementary pair of length $N = 4$ and apply it to a 8×8 WH matrix.

Let S_1=[1 1 1 -1] and S_2=[1 1 -1 1] be the complementary pair. We can generate a mate pair by: $(S_1, S_2) \longleftrightarrow (\bar{S_2}^*, -\bar{S_1}^*)$,

where '-' indicates reverse of a sequence and '*' indicates complex conjugate. For easy understanding, we can call $S_1 = c_0$, $S_2 = s_0$, $\bar{S_2}^* = c_1$ and $-\bar{S_1}^* = s_1$.

From this construction, a family size of $4P$ is generated, but only set of P LS sequences exhibit IFW with each other. Out of a family size of $4P$, we have chosen a set of 8 sequences for the 8 user case, and 15 sequences, from two sets of 8 sequences for the 15 user case.

TABLE I

MEAN SQUARE CROSS-CORRELATION (MSCC) OF 15 SEQUENCES FOR THREE DIFFERENT FAMILIES

Sequence family	$\tau = 1 - N$ to $N - 1$	$\tau = -3$ to 3
Oppermann L=31, M=15 (m,n)=(1.0038,1.2971)	0.1983	0.0122
Walsh Hadamard L=32, M=15	0.5199	0.0709
LS (N, P, W_0)=$(4, 8, 3)$ L=38, M=15	0.933	0.1155

TABLE II

MEAN SQUARE CROSS-CORRELATION (MSCC) OF 8 SEQUENCES FOR THREE DIFFERENT FAMILIES

Sequence family	τ=-3 to 3	τ=-4 to 4	τ=-6 to 6
Oppermann L=31, M=8 (m, n)=$(1.0038, 1.2971)$	0.0101	0.0148	0.0219
Walsh Hadamard L=32, M=8	0.0229	0.0285	0.0444
LS (N, P, W_0)=$(4, 8, 3)$ L=38, M=8	0	0.2162	0.2294

C) Optimized Subsets of Sequences

It is quite common for many sequence families that the number of sequences in a set are determined by the sequence length [9]. Rather than choosing sequences randomly within a set, if a wireless system is lightly loaded then it is better to choose subset of sequences optimized for cross-correlation. These optimized subsets exhibit significantly less cross-correlations values, as compared to their corresponding sequence sets. In [9], a branch and bound algorithm is presented to generate optimal subsets of different sequences. Based on [9], we have generated the optimized sequence subsets and then calculated their MSCC and out of phase MSAC values. Table. 1, present 15 sequence subset of WH and Oppermann sequences and compare them with 15 sequences from two sets of LS sequences. In Table. 1, average MSCC values of three different sequence families are compared for two different chip delay values. In Table. 2, optimized subsets comprising of 8 sequences are compared for three different chip delay values. It can be seen from Table. 1, that for 15 user system, optimized subsets of WH and Oppermann sequences outperform LS sequences for QS-CDMA. It can also be seen from Table. 1, that this performance is achieved, when quasi-synchronization is kept within IFW of LS sequences. In Table. 2, LS sequence set size of 8, outperform optimized subsets of WH and Oppermann sequences, when chip delays are kept within IFW of LS sequences. But for 1-3 chip delays outside the IFW, optimized subsets of WH and Oppermann sequences outperform LS sequence set. It is significantly important to note from Table. 2, that for LS sequences, a single chip delay outside IFW results in significant deterioration of performance, in terms of cross-correlation values. In Table. 3, out-of-phase MSAC values of three sequence families are shown. It is evident form Table. 3, that in terms of out-of-phase MSAC values, LS sequence sets outperform optimized subsets of WH and Oppermann sequences.

TABLE III

OUT-OF-PHASE APERIODIC MEAN SQUARE AUTO-CORRELATION OF 15 SEQUENCES FOR THREE DIFFERENT FAMILIES

Sequence family	τ=-3 to 3	τ=-4 to 4	τ=-6 to 6
Oppermann L=31, M=8 (m, n)=$(1.0038, 1.2971)$	5.1874	6.6450	9.1807
Walsh Hadamard L=32, M=8	1.8784	2.6096	3.3184
LS (N, P, W_0)=$(4, 8, 3)$ L=38, M=8	0	0.0020	0.9488

III. LS SEQUENCES AND CAPACITY

Sequences employed in traditional CDMA have major impact on Signal to Interference Ratio (SIR) and spectral efficiency (C/B) of the system. For a traditional CDMA SIR is given as [10]:

$$SIR = \frac{P_u G}{(N-1)P_u}, \tag{9}$$

where G is the processing gain, P_u is the user signal power and $N - 1$ is the number of interfering users. Similarly, (C/B) of a traditional CDMA system is given as [10]:

$$C/B = \frac{log_2 M}{SIR}, \tag{10}$$

where M represents M-ary modulation. Cross-correlation properties of sequences effect Multiple Access Interference (MAI). It is evident from Eq. 9, that MAI from other users is $(N - 1)$, and it can significantly reduce system capacity and spectral efficiency. Therefore, for conventional CDMA, cross-correlation properties of sequences are of great significance. Similarly, for QS-CDMA, SIR is given as [10]:

$$SIR = \frac{P_u G}{\alpha(N-1)P_u}, \tag{11}$$

where α represent remaining interference ratio with the use of ZCZ sequences. Similarly, C/B is given as [10]:

$$C/B = \frac{log_2 M}{\alpha SIR}. \tag{12}$$

The above two equations clearly reflect that for QS-CDMA also, cross-correlation properties of sequences employed, play an important role. It can be seen from Eqs. 11 and 12, that if $\alpha < 1$, then QS-CDMA have greater spectral efficiency then traditional CDMA. In [10], it is assumed that for ZCZ sequences α is typically < 1. However, existing LS sequence family set sizes, exhibiting IFW are bounded [4]. For example, when we construct a LS sequence family of spreading factor 38, we have four sets of 8 sequences exhibiting IFW of 3 chips. As shown in Eq. 1, there is also a bound on the length of IFW with respect to the family size and the length of the sequence. It can be seen from Eq. 1, that the family size of ZCZ sequences can never exceed the length of the sequence. Due to these limitations on LS sequence

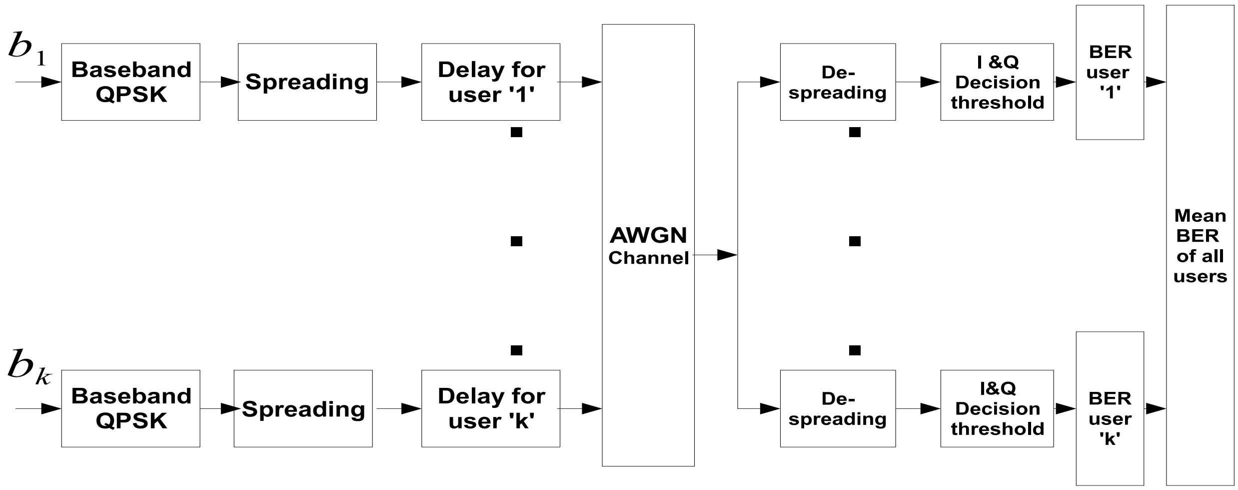

Fig. 1. Simulation model for QS-CDMA. In case of LS sequences, additional blocks of gap insertion and gap removal are included at the transmitters and receivers respectively.

families, when a family size of 32 is used for 15 users in QS-CDMA, it performs worse than optimized subsets of WH and Oppermann sequnces in terms of average MSCC values. In Table. 1, it is shown that for QS-CDMA having chip delays within IFW of LS sequences, optimized subsets of WH and Oppermann sequences outperform LS sequences. MAI of LS sequences in this case is greater than MAI suffered by the optimized subsets of WH and Oppermann sequences. As shown in Eqs. 9 and 11, $(N-1)P_u$ and $\alpha(N-1)P_u$ represent effect of MAI on traditional CDMA and QS-CDMA systems respectively. For the LS sequence family used in this paper, it can be seen from Table. 1, that $\alpha(N-1)P_u$ $> (N-1)P_u$. For the family size 32 and 15 users case, $\alpha > 1$ for the existing LS sequences, when compared with optimized subsets of Oppermann and WH sequences. In Table. 2, sequences for 8 users are compared. It can be seen that for 3-chip delay QS-CDMA, LS sequences outperform optimized subsets of WH and Oppermann sequences, due to the presence of IFW of length 3. But when chip delays are slightly increased outside IFW, this performance of LS sequences deteriorates, and they perform worse than the optimized subsets. It is evident from Table. 2, that even for a single chip delay outside IFW, optimized subsets outperform LS sequences in terms of MSCC and this performance of LS sequences further degrades with the introduction of further delays outside IFW.

IV. Simulation Model and Results

Uplink quasi-synchronous transmitters and receivers for k users operating in an AWGN channel are shown in Fig. 1. It is important to note here that when LS sequences are used for transmission, then we need additional blocks of gap insertion and gap removal at the transmitters and receivers respectively. The simulation model involves generation of data by k users. The kth user data is modulated by the QPSK modulator. This results in the generation of complex baseband symbols, which are subjected to spreading by spreading sequences. If LS sequences are employed for spreading, then appropriate gaps are inserted at this stage, otherwise this step is skipped. Finally, appropriate chip delays are introduced. It is important to note that in the simulation model, users are subjected to static delays rather than dynamic delays.

The k users baseband signal is transmitted through AWGN channel, and on the receiver side signals are de-spread and detected. BER for each user is then evaluated within each Signal-to-Noise Ratio (SNR), and finally average BER for all users is calculated. From our simulation results, for 8 users system, it can be seen from Fig. 2, that the subset of Oppermann sequences give comparable performance with respect to LS sequences. This performance is achieved, when a single user suffers 1 chip delay outside the IFW of LS sequences. In Fig. 3, performance of LS sequences is similar to the subset of WH sequences, and subset of Opperman sequences outperform LS sequences. This performance is achieved when two user delays are kept, 2 and 3 chips outside the IFW of LS sequences. For 15 users case, subsets of both Oppermann and WH sequences outperform LS sequences, even when the chip delays are kept within the IFW of LS sequences.

V. Conclusion

In terms of auto-correlation properties, LS sequences outperform Oppermann and WH sequences. But in terms of cross-correlation properties, there exist traditional sequence subsets, that give either comparable performance, or outperform well known construction of LS sequences.

When simulated for QS-CDMA system under consideration, BER performance of LS sequences for 15 users degrades, as compared to the performance of Oppermann and WH sequence subsets. It is observed, that this performance degradation is due to high cross-correlation values of LS sequences outside their IFW. It is also shown, that this performance degradation is also due to the presence of bounds on LS sequence families. BER analysis for 8 users show, that for a single user having a single chip delay outside IFW, Oppermann sequence subset give comparable performance. It is also shown that for 2-3 chip delays outside IFW, Oppermann sequence subset outperform LS sequence set.

It is therefore concluded, that for lightly loaded and short code QS-CDMA systems, better constructions of LS sequences are required, when cross-correlation values are considered.

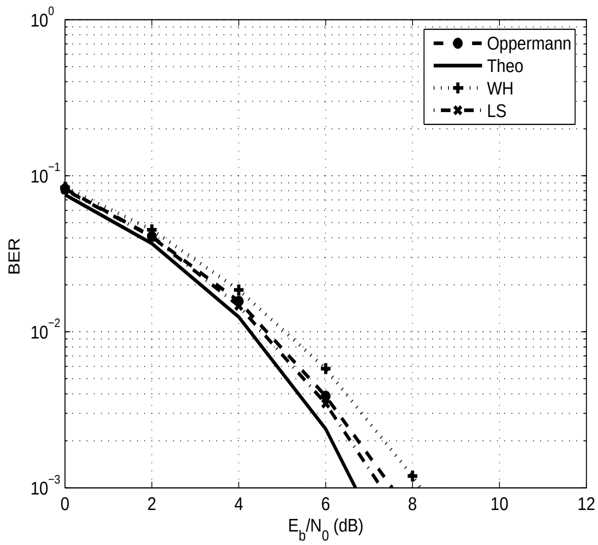

Fig. 2. BER comparison for 8 users in AWGN channel, with a single user having 1 chip delay outside the IFW of LS sequences.

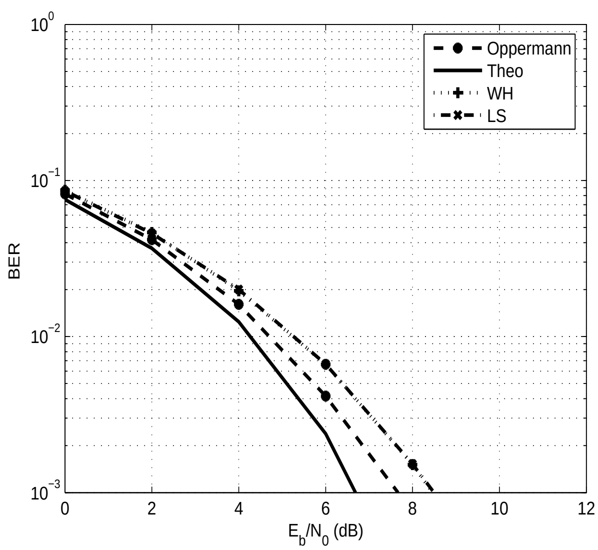

Fig. 3. BER comparison for 8 users in AWGN channel, with 2 users having 2 to 3 chip delays outside the IFW of LS sequences.

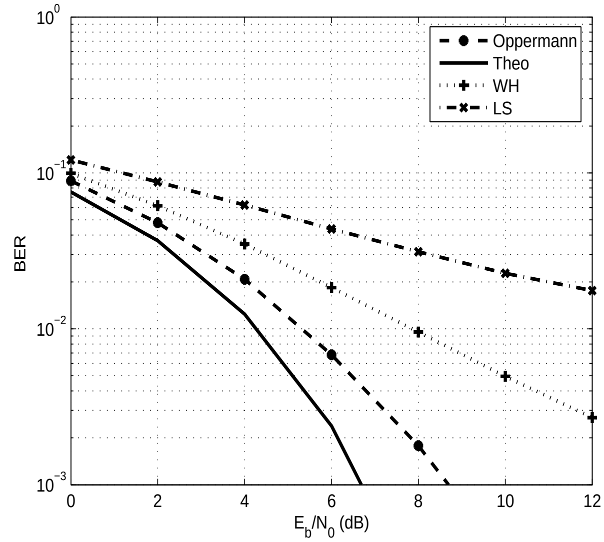

Fig. 4. BER comparison for 15 users in AWGN channel, with chip delays within the IFW of LS sequences.

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