

# Sensitivity analysis of on-chip passive structures

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**Abstract**—The integration of analog and digital sub-systems on a single die necessitates the use of highly accurate but compact models of electromagnetic effects in integrated passive structures for the successful design of next generation Integrated Circuits (IC). This paper examines the use of the Finite Integral Technique (FIT) and Taylor Series expansion to extract parametric compact models of integrated passive structures. A software prototype implementing these methods is then used to investigate the sensitivity of several standard benchmark structures to environmental variations.

**Keywords**—Coupled Problems, Integrated Circuit, Simulation Software, Sensitivity Analysis, Taylor Series

## I. INTRODUCTION

Driven by consumer demand, the trend in today's semiconductor industry is not only towards more compact integrated circuits (IC) [1], but also more complex functionality. This leads to the integration of analog (RF) and digital subsystems on a single silicon chip. Furthermore, due to greater demand for bandwidth by multimedia applications, the operating frequency of these integrated circuits is also increasing. At gigahertz frequency of operation however, electromagnetic (EM) signal propagation on interconnect structures are very susceptible to changes in the properties of the environment.

Because of this, accurate parametric compact models are needed for the successful design of next generation integrated mixed signal silicon. Furthermore, the area of nanoElectronic Design Automation (nEDA) has been identified by the European Nanoelectronics Initiative Advisory Council (ENIAC) as a key thrust of its Strategic Research Agenda (SRA) to spearhead development of design tools that address these challenges. The CHAMELEON-RF (CHRF) project was hence initiated to develop tools and methodologies for comprehensive high accuracy modeling of on-chip electromagnetic effects considering environmental variability [2, 3]. The CHRF software prototype enables the extraction of parametric compact models (SPICE compliant) of passive integrated structures for sensitivity analysis of material variation in the environment. Furthermore, parametric models enable the simulation of sensitivity to variation without recomputing new models for the whole design.

In this paper, we first present in section 2, the key points of the Finite Integration Technique (FIT) [9] discretization method and parametric model extraction using the Taylor Series (TS) expansion [7]. Section 3 then presents results from sensitivity analysis of some standard benchmark structures using the CHRF software prototype developed in LMN.

## II. PARAMETRIC COMPACT MODEL EXTRACTION

The proposed approach to modeling passive integrated structures ICs is based on three important stages, with the

corresponding numerical methods associated for the analysis of the model extracted at each stage: continuous model, discrete model and a reduced model.

The first stage in the modeling process is the extraction of the continuous model which describes the EM field problem for passive components after Domain Decomposition (DD). It is a system of Partial Differential Equations (PDE) extracted based on Maxwell equations (1-4)

$$\text{curl } \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}; \quad \text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1-2)$$

$$\text{div } \mathbf{D} = \rho; \quad \text{div } \mathbf{B} = 0 \quad (3-4)$$

and boundary conditions (5-8) associated with the concept of Electromagnetic Circuit Elements (EMCE) [4, 5].

$$\mathbf{n} \text{ curl } \mathbf{E}(P,t) = 0 \quad \text{for } \forall P \in \Sigma - S_k'' \quad (5)$$

$$\mathbf{n} \text{ curl } \mathbf{H}(P,t) = 0 \quad \text{for } \forall P \in \Sigma - S_k' \quad (6)$$

$$\mathbf{n} \times \mathbf{E}(P,t) = 0 \quad \text{for } \forall P \in \cup S_k' \quad (7)$$

$$\mathbf{n} \times \mathbf{H}(P,t) = 0 \quad \text{for } \forall P \in \cup S_k'' \quad (8)$$

In the second stage of the modeling process, for numerical analysis of fields in the time domain the FIT is used for discretization of the continuous model [9]. The FIT numerical method aims to describe the model in the finite space of Differential Algebraic Equations (DAE). The basic concept of the classical FIT is the spatial discretization by DAEs of (1-4) into the Maxwell Grid Equations (MGE) (9-13) with the inclusion of Hodge equations which describe the material behavior (14-16). The FIT operates with global variables – Degrees of Freedom (DOFs) – namely the voltages and fluxes on grid elements instead of local field elements. Furthermore, MGEs which have no discretization errors, are metric-free, sparse, mimetic and form a conservative system of DAEs, without spurious modes.

As a result, the sparse state-space representation of the component is obtained, which is a Linear Time Invariant system (LTI) of equations (17-18).

$$\begin{cases} (2) \\ (4) \end{cases} \Rightarrow \begin{cases} \oint \mathbf{E} d\mathbf{r} = -\iint \frac{\partial \mathbf{B}}{\partial t} d\mathbf{A} \\ \iint \mathbf{B} d\mathbf{A} = 0 \end{cases} \Rightarrow \begin{cases} \mathbf{C}'\mathbf{v} = -\frac{d\boldsymbol{\varphi}}{dt} \\ \mathbf{D}'\boldsymbol{\varphi} = 0 \end{cases} \quad (9-13)$$

$$\begin{cases} (1) \\ (3) \end{cases} \Rightarrow \begin{cases} \oint \mathbf{H} d\mathbf{r} = \iint (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) d\mathbf{A} \\ \iint \mathbf{D} d\mathbf{A} = \iint \rho dv \end{cases} \Rightarrow \begin{cases} \mathbf{C}'\mathbf{u} = \mathbf{i} + \frac{d\boldsymbol{\psi}}{dt} \\ \mathbf{D}\boldsymbol{\psi} = \mathbf{q} \end{cases}$$

$$\Rightarrow \text{div} \mathbf{J} = -\frac{\partial \rho}{\partial t} \Rightarrow \iint \mathbf{J} d\mathbf{A} = -\iint \frac{\partial \rho}{\partial t} dv \Rightarrow \mathbf{D}\mathbf{i} = -\frac{d\mathbf{q}}{dt}$$

$$\begin{cases} \mathbf{B} = \mu \mathbf{H} \\ \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{J} = \sigma \mathbf{E} \end{cases} \Rightarrow \begin{cases} \boldsymbol{\varphi} = \mathbf{M}_\mu \mathbf{u} = \mathbf{M}_\nu^{-1} \mathbf{u} \\ \boldsymbol{\psi} = \mathbf{M}_\epsilon \mathbf{v} \\ \mathbf{i} = \mathbf{M}_\sigma \mathbf{v} \end{cases} \quad (14-16)$$

$$\begin{cases} \mathbf{E} \frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} \end{cases} \quad (17-18)$$

Matrices  $\mathbf{E}$ ,  $\mathbf{A}$ ,  $\mathbf{C}$  consist of blocks of the Hodge matrices and topological matrices resulting from EMCE and boundary conditions concepts [6, 7]. The Taylor Series (TS) first order sensitivities of these matrices are essential for the parameter variability analysis.

The first order truncation of the TS is expressed as (19), considering  $F$  as a device characteristic, dependent on the design parameters ( $p_1, p_2, \dots, p_n$ ), which nominal values are  $\mathbf{p}_0 = (p_{01}, p_{02}, \dots, p_{0n})$ . The partial derivatives of device characteristics (20) computed for the nominal values  $\mathbf{p}_0$  are first order sensitivities.

$$F_1(p_1, p_2, \dots, p_n) = F(p_{01}, p_{02}, \dots, p_{0n}) + S_{p_1}(p_1 - p_{01}) + S_{p_2}(p_2 - p_{02}) + \dots + S_{p_n}(p_n - p_{0n}) \quad (19)$$

$$S_{p_1} = \frac{\partial F}{\partial p_1}, S_{p_2} = \frac{\partial F}{\partial p_2}, \dots, S_{p_n} = \frac{\partial F}{\partial p_n} \quad (20)$$

Using the TS for state-space matrices (17-18) the output can be defined as (21), where (22-23) are the sensitivities of state representation matrices and  $\mathbf{E}_0 = \mathbf{E}(\mathbf{p}_0)$ ,  $\mathbf{A}_0 = \mathbf{A}(\mathbf{p}_0)$  the matrices computed for the nominal values of the parameters  $\mathbf{p}_0$ .

$$\mathbf{y}(\mathbf{p}) = \mathbf{C}(j\omega \mathbf{E}_{\text{TS}} - \mathbf{A}_{\text{TS}})^{-1} \mathbf{B}\mathbf{u} \quad (21)$$

$$\mathbf{E}_{\text{TS}} = \mathbf{E}_0 + \frac{\partial \mathbf{E}}{\partial p}(p - p_0) \quad (22)$$

$$\mathbf{A}_{\text{TS}} = \mathbf{A}_0 + \frac{\partial \mathbf{A}}{\partial p}(p - p_0) \quad (23)$$

The compact modeling of EM effects in ICs starts initially from a full-wave EM field problem description. The crucial step in the modeling process is the discretization of the continuous model which is effectively handled by FIT. Furthermore, a parametric model can be obtained using the TS expansion. As a final result, a dimensionally- and materially-parameterized model is obtained that enables the computationally efficient simulation of passive structures.

### III. SENSITIVITY ANALYSIS RESULTS

Benchmark test structures of device coupling structures were designed and fabricated on a 0.35 $\mu\text{m}$  BiCMOS process

and measured at an industrial partner - Austria Micro Systems (AMS). The modeling methods described in the previous section were validated by comparing simulation results with measured data for fabricated benchmarks. The Chamy prototype code developed by LMN within the CHRf project was used in the simulation and validation of these benchmark structures. Finally, sensitivity analysis of environmental variation on selected test structures was performed. The results from the RFPAD test structure is presented below.

The benchmark RFPAD is a metal stack placed over a 3  $\mu\text{m}$  thick nwell layer that acts as shielding (Fig.1). RF pads are a widely used structure in the semiconductor industry for off-chip packaging connections via wire-bond leads or flip chip technology and also functions as a filter for high frequency components of the signal.

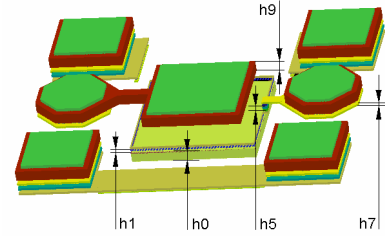


Fig.1. A 3D view of RFPAD test structure with height variation of different non planarized layers ( $h_x$ ).

The S parameter plots for the simulation using the Chamy prototype software are validated against measurement as shown in Fig.2. Together with this, the Chamy software also enables the sensitivity analysis of the structure to be performed with respect to variations in the fabrication process.

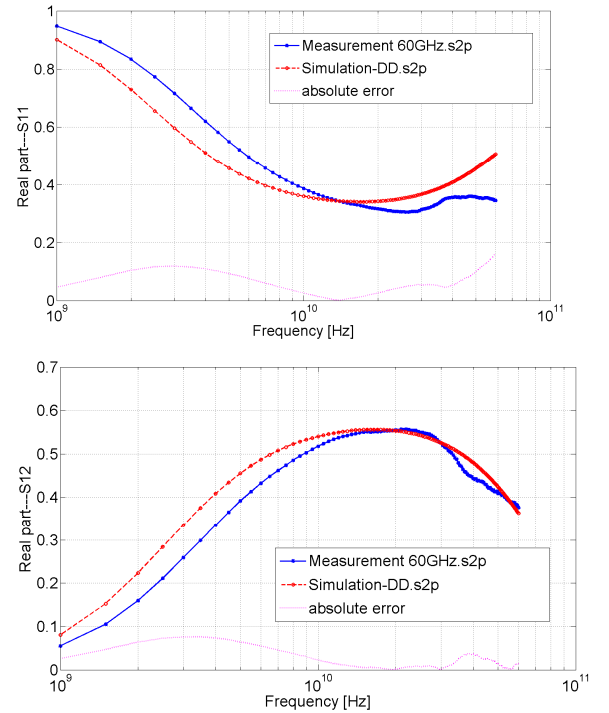


Fig. 2. Measured and simulated S parameters of the RFPAD benchmark.

One important parameter is the height variation of various deposition layers. For a particular semiconductor fabrication process, layer height data are process parameter values that are measured during production using various electrical and optical methods. Apart from the nominal values, data on the minimum and maximum limits of deviation from the nominal values can also be obtained. Using the data from the industrial partner site, the sensitivity of the RFPAD S parameter characteristics to layer height variation was analyzed. Several layer heights ( $h_x$ ) such as the nwell layer and metal layers corresponding to the different layers were analyzed.

Amongst all the various heights attempted, the variation of the nwell diffusion layer ( $h_0$ ) had the most impact on the S21 characteristics of the RFPAD which indicates the importance of the nwell shielding layer. Figure 3 presents the relative variation of output 21 influenced by 20% variation of layer heights  $h_1$ ,  $h_5$ ,  $h_7$  and  $h_9$ ; the impact of these parameters can be disregarded in contrast to the impact of  $h_0$ . Figure 4 illustrates the percentage change in S21 for +20 to -20% variation in the depth of the nwell diffusion layer. A notable result is that for frequencies of operation between 20 – 25 GHz, the S21 parameters are relatively independent of variations in nwell depth for this particular process.

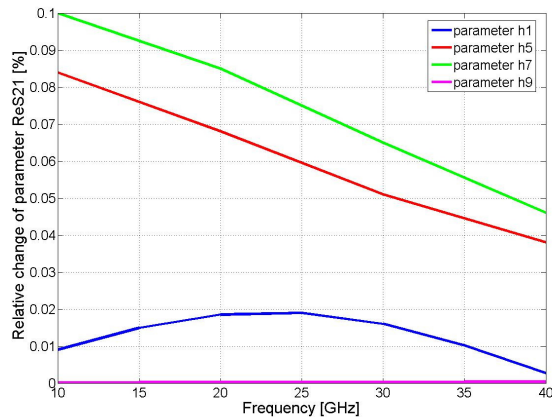


Fig. 3. Relative variation of real part of output 21 in frequency domain, influenced by 20% change of parameters  $h_1$ ,  $h_5$ ,  $h_7$ ,  $h_9$

#### IV. CONCLUSIONS

Denser integration of devices inevitably leads to greater impact of EM coupling between circuits. The use of parametric compact models extracted using the TS expansion has been shown to be an effective method in managing the complexity of accurately modeling these effects. Moreover, the parameterized model obtained with the applied FIT+TS methodology enables the computationally efficient simulation.

The sensitivity analysis allows investigating the frequency response variation due to geometry of the structure. By separation of the most relevant parameter (in context of the greatest influence on the model characteristics) the fabrication process effort can be properly planned.

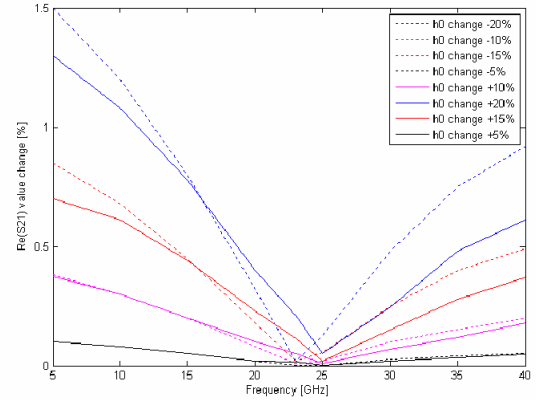


Fig. 4. Percentage change of RFPAD  $\text{Re}(S_{21})$  due to variation in nwell depth ( $h_0$ ).

#### ACKNOWLEDGEMENT

The authors would like to gratefully acknowledge support and insightful discussion with Prof. Daniel Ioan, Asc.Prof. Gabriela Ciuprina and the LMN Chameleon project team as well as thank Austria Micro Systems for the experimental data. The authors would also like to acknowledge the financial support offered by the European Commission under the Marie Curie Fellowship (FP6/EST3) programme.

The views stated herein reflect only the authors' views and the European Commission is not liable for any use that may be made of the information contained.

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